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Unsettled Problems of Second-Order Quantum Theories of Elementary Particles

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ABSTRACT

The physical community agrees that the variational principle is a cornerstone of a quantum fields theory (QFT) of an elementary particle. This approach examines the variation of the action of a Lagrangian density whose form is $S = \int d^4x \mathcal{L}(\psi, \psi_{,\mu})$. The dimension of the action S and d^4x prove that the quantum function ψ of any specific Lagrangian density $\mathcal{L}(\psi, \psi_{,\mu})$ has a definite dimension. This evidence determines the results of new consistency tests of QFTs. This work applies these tests to several kinds of quantum functions of a QFT of elementary particles. It proves that coherent results are derived from the standard form of quantum electrodynamics which depends on the Dirac linear equation of a massive charged particle and Maxwell theory of the electromagnetic fields. In contrast, contradictions stem from second-order quantum theories of an elementary particle, such as the Klein-Gordon equation and the electroweak theory of the $W^{\pm}$ boson. An observation of the literature that discusses the latter theories indicates that they do not settle the above-mentioned crucial problems. This issue supports the main results of this work.

Keywords: Quantum fields theories; the variational principle; the dirac equation; second-order quantum equations.

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1 INTRODUCTION

Dimension is a fundamental property of any physical quantity. Every person who has studied physics becomes acquainted with this concept at the very early period of his studies. This work discusses quantum field theories (QFT) of elementary particles and shows new features of the dimension of quantum functions. The standard system of units, where $\hbar = c = 1$ is used. Thus, one independent dimension is relevant and a power of the length $[L^n]$ denotes it.

A well-known property of quantum mechanics is the particle's density

$$\rho = \psi^* \psi \tag{1}$$

(see [1], p. 22; [2], p. 37). This expression means that the dimension of the Schroedinger function ψ is $[L^{-3/2}]$. As is well known, the dimension of a classical quantity takes an integral power. It means that the quantum function ψ has no classical analog. This issue may be regarded as an awkward property. However, it does not deny the acceptability of the Schroedinger quantum theory, because the ordinary form of the expectation value of a physical variable is

$$< O >= \int \psi^* \hat{O} \psi \, d^3r, \tag{2}$$

where $\hat{O}$ denotes the operator that represents the variable O. Equation (2) proves the importance of the dimension $[L^{-3/2}]$ of the quantum function. Indeed, *it ensures that the dimension of the operator of a given variable equals the dimension of the variable's expectation value.* This issue is a key element of the discussion of this work, that analyzes several aspects of QFT of elementary particles.

The people who have constructed quantum mechanics recognized the need for the correspondence between the classical limit of quantum mechanics and classical physics. This issue is documented in textbooks: "Classical mechanics must therefore be a limiting case of quantum mechanics" (see [3], p. 84). Similar statements can also be found in other textbooks (see e.g. [1], pp. 137, 138; [2], p. 3). Hence, the mathematically real value of a classical variable entails that the operator $\hat{O}$ of (2) should be Hermitian.

The relevance of quantum mechanics to QFT is clearly stated in Weinberg's textbook (see [4], p. 49):

> *"First, some good news: quantum field theory is based on the same quantum mechanics that was invented by Schroedinger, Heisenberg, Pauli, Born, and others in 1925-26, and has been used ever since in atomic, molecular, nuclear and condensed matter physics."*

These arguments mean that quantities of QFT are related to corresponding quantities of quantum mechanics as well as to those of classical physics. Below, this relationship is called the *generalized correspondence principle.* Few pages at the beginning of Rohrlich's book [5] discuss this topic. The rest of this work proves that an analysis of the dimension of well-known quantum functions yields far-reaching consequences. In so doing, this work may help closing the gap created from the negligence of this topic in the present literature.

Relativistic expressions are written in the standard notation. The Minkowski metric $g_{\mu\nu}$ is diagonal and its entries are (1,-1,-1,-1). Greek indices run from 0 to 3 and Latin indices run from 1 to 3. Section 2 describes several constraints that are imposed on an acceptable quantum theory. These constraints are utilized by the analysis. Section 3 examines theories of the Standard Model (SM). It proves that quantum electrodynamics (QED) has a coherent structure while inherent contradictions are found in second-order quantum theories of an elementary charged particle, such as the Klein-Gordon equation and the electroweak equation of the $W^{\pm}$ bosons. A discussion of the results is carried out in section 4. Section 5 proves that the energy-momentum tensor of second-order quantum theories violates fundamental physical principles. In particular, the Higgs boson theory is unacceptable. The last section summarizes this work.

2 RELEVANT CONSTRAINTS

Physics is a mature science that has already several well-established principles, and any specific theory must abide by them. Some of these principles yield constraints that apply to every quantum theory of an elementary particle. The list below shows constraints that are used by the mathematical analysis of this work.

[label=CON.0] The primary role of special relativity means that a quantum theory of an elementary particle should have a relativistic structure [4, 6]. Therefore, an acceptable QFT should take a relativistic covariant form. A QFT of a given elementary particle should be derivable from the variational principle that uses a Lagrangian density of the form

$$\mathcal{L}(\psi, \psi_{,\mu}). \tag{3}$$

The quantum function $\psi(t, \boldsymbol{x})$ is the generalized coordinate of the Lagrangian density $\mathcal{L}$. This point is agreed by the physical community. For example, Weinberg states: "all field theories used in current theories of elementary particles have Lagrangians of this form" (see [4], p. 300). The variational principle says that the time evolution of the system is determined by equations that are derived from a minimum of this action

$$S = \int d^4x \mathcal{L}(\psi, \psi_{,\mu}). \tag{4}$$

This principle yields the system's equations of motion

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}} \right) - \frac{\partial \mathcal{L}}{\partial \psi} = 0, \tag{5}$$

which are called the Euler-Lagrange equations (see [4], p. 300; [6], p. 16; [7], p. 14). In the unit system used herein the action is dimensionless. Therefore, the four integrals of (4) prove that the dimension of the Lagrangian density is $[L^{-4}]$. Moreover, d^4x is a Lorentz scalar. Hence, the Lagrangian density must be a Lorentz scalar. A Lorentz scalar Lagrangian density ensures that the theory abides by special relativity (see [6], p. 35). The dimension of the Lagrangian density determines the dimension of every quantum function of any given QFT of an elementary particle. This property is a primary element of the analysis that is described below. Maxwellian electrodynamics is a theory of *two* physical objects: a massive charged particle and electromagnetic fields. QED is a theory of these objects, and its Lagrangian density is

$$\mathcal{L}_{QED} = \bar{\psi}[\gamma^\mu i \partial_\mu - m]\psi - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \bar{\psi} e \gamma^\mu A_\mu \psi. \tag{6}$$

(see e.g. [6], p. 78; [7], p. 84). Here ψ is the quantum function of a charged Dirac particle; each γ^μ is one of the four Dirac matrices that take the representation of [8]; $F^{\mu\nu}$ is the tensor of the electromagnetic fields; A_μ is the electromagnetic 4-potential and its components are regarded as the generalized coordinates of the electromagnetic fields; $\bar{\psi} = \psi^\dagger \gamma^0$. The quantities inside the square brackets of (6) are the kinetic and the mass terms of the Dirac particle, respectively; the second term of (6) represents the electromagnetic fields; the last term represents the interaction between the electromagnetic fields and the charged particle. The QED Lagrangian density (6) yields the first order Dirac equation of the massive spin-1/2 charged particle *and* Maxwell equations of the electromagnetic fields. The QED Lagrangian density (6) is an extremely important and successful expression. For example, Peskin and Schroeder state: "That such a simple Lagrangian can account for nearly all observed phenomena from macroscopic scales down to 10^{-13} cm is rather astonishing" (see [6], p. 78). Maxwell equations describe the time evolution of the electromagnetic fields

$$F^{\mu\nu}_{,\nu} = -4\pi j^\mu; \quad F^{*\mu\nu}_{,\nu} = 0. \tag{7}$$

Here $F^{*\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ and $\epsilon^{\alpha\beta\gamma\delta}$ is the completely antisymmetric unit tensor of the fourth rank. The first equation of (7) is called the inhomogeneous Maxwell equation, and it depends on the 4-current j^μ of the charged particle. The second equation of (7) is called the homogeneous Maxwell equation. As stated above, the QED Lagrangian density (6) yields the Dirac equation of a charged spin-1/2 massive particle *and* Maxwell equations of the electromagnetic fields. There is another aspect of the interrelations between the theories of these physical entities. It turns out that the inhomogeneous Maxwell equation of the electromagnetic fields *imposes a constraint on the 4-current of every charged particle.* Thus, taking the 4-derivative of the inhomogeneous Maxwell equation (7), one obtains

$$F^{\mu\nu}_{,\nu,\mu} = 0 \quad \rightarrow \quad j^\mu_{,\mu} = 0, \tag{8}$$

where the null result is obtained from the antisymmetry of $F^{\mu\nu}$. Eq. (8) of the 4-current is called the continuity equation (see [8], p. 24; [9], pp. 76, 77). The 4-current is a well-known relativistic 4-vector and its component j^0 is the charge density. Sometimes a multiplication factor normalizes j^μ so that

$$\int j^0\, d^3r = 1, \tag{9}$$

which means that the charge density is ej^0. The continuity equation is the mathematical expression of the experimentally confirmed effects of charge conservation and its continuous motion.

These arguments prove that the 4-current of a massive charged particle is a crucial theoretical quantity. It has these properties:

[label=P.0] The electromagnetic fields are mathematically real quantities. Hence, Maxwell equations (7) prove that the 4-current is a mathematically real 4-vector. Relation (9) means that the dimension of the 4-current is $[L^{-3}]$. The 4-current satisfies the continuity equation (8).

The Noether theorem is an important element of the theoretical structure of QFT. Referring to the 4-current issue, this theorem takes advantage of the variational principle and provides a standard expression for a conserved 4-current

$$j^\mu = a\frac{\partial \mathcal{L}}{\partial \psi_{,\mu}}\psi, \tag{10}$$

where a is an appropriate numerical coefficient (see [10], pp. 314, 315). j^0 is the particle's density. Hence a is fixed so that $\int j^0 d^3x = 1$.

2. An important property of the Noether theorem (10) says that a term of the Lagrangian density that is independent of a derivative of the quantum function $\psi_{,\mu}$ makes a null contribution to the form of the 4-current.

The 4-current of a quantum theory of a charged quantum particle plays a key role in the following examination.

3 MATHEMATICAL ANALYSIS

The present QFT structure of electrodynamics discusses *one unique kind of electromagnetic fields* $F^{\mu\nu}$ *that interact with one unique kind of electric charge* e. Experiment proves that the particle form of electromagnetic fields is the massless photon, while the electric charge is carried by a massive particle. The SM describes *two distinct families of theories of elementary massive charged particles:* The first-order Dirac theory of a spin-1/2 particle and second-order equations of integral spin particles, like the mathematically complex Klein-Gordon (KG) equation and the electroweak equation of the $W^{\pm}$. It is explained in the previous section that the electromagnetic fields impose constraints on the 4-current of a charged particle. This work examines the compatibility of each kind of the SM theories of an elementary charged particle with these constraints.

Experiments support the concept of one kind of electric charge. Take for example this decay channel: $W^- \rightarrow e\bar{\nu}$ [11]. The SM says that the second-order electroweak theory describes the W^- whereas the Dirac QED theory (6) describes the electron. Hence, the above-mentioned W^- decay channel and the charge conservation law say that the electric charge of the W^- is the same as that of the electron.

One objective of this work is to prove that unsettled problems emerge from an attempt to construct an electromagnetic theory of a second-order quantum equation that is analogous to the QED Lagrangian density of (6). Special attention is devoted to the dimension of physical quantities and to properties 1 - 3 of a coherent 4-current that are mentioned near the end of the previous section.

3.1 The Coherent Structure of QED

An examination of the ordinary QED structure that depends on the first-order Dirac equation clarifies the problem. Here the Noether expression for the conserved 4-current (10) yields

$$j^{\mu} = \bar{\psi}\gamma^{\mu}\psi \tag{11}$$

(see [6], pp. 14, 15; [8], pp. 23, 24). The following items explain why this QED 4-current satisfies the requirements 1 - 3:

- The Noether expression (11) is a 4-vector (see [8], p. 24). The definition $\bar{\psi} = \psi^{\dagger}\gamma^0$ and the Dirac γ matrices prove that components of this 4-vector are

  $$\bar{\psi}\gamma^0\psi = \psi^{\dagger}\gamma^0\gamma^0\psi = \psi^{\dagger}\psi \tag{12}$$

  and

  $$\bar{\psi}\gamma^i\psi = \psi^{\dagger}\gamma^0\gamma^i\psi = \psi^{\dagger}\alpha_i\psi, \tag{13}$$

  (see [8], pp. 8, 17). Here $\psi^{\dagger}\psi$ is mathematically real and the Dirac α_i are Hermitian matrices (see [8], p. 8). Hence, their expectation values are mathematically real numbers and (11) is compatible with 1.
- The dimension of the Lagrangian density is $[L^{-4}]$. Hence, the QED Lagrangian density (6) proves that the dimension of the Dirac quantum function ψ is $[L^{-3/2}]$, and entries of the γ matrices are pure numbers. It follows that the dimension of the Dirac 4-current (11) is $[L^{-3}]$. It is explained in the introduction, after (2), that the dimension $[L^{-3/2}]$ of the quantum function is an adequate property and it is compatible with 2.
- The Dirac 4-current (11) satisfies the continuity equation (see [8], p. 9), which means that it is compatible with 3.

These issues show the coherent structure of QED, where the Dirac equation of a charged massive spin-1/2 particle and Maxwell equation of the electromagnetic fields describe the time evolution of the system.

Concerning the QED Lagrangian density, the components of the 4-potential A_{μ} are the generalized coordinates of the electromagnetic fields (see the footnote on p. 72 of [9]; [12], p. 596). The electromagnetic fields are the derivatives

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \tag{14}$$

(see [9], p. 65; [12], p. 596). A significant attribute of this system is that Maxwell equations (7) are independent of the 4-potential A_{μ}. This property is called gauge invariance. The first-order derivative of the Euler-Lagrange equation (5) with respect to the electromagnetic generalized coordinates A_{μ} yields this important conclusion:

Maxwellian Gauge Invariance Requirement: Maxwellian gauge invariance requires that the Lagrangian density of electromagnetic fields should not contain a term that has a factor where the power of A_μ is greater than 1.

A straightforward observation of the QED Lagrangian density (6) proves that the Dirac theory abides by the Maxwellian Gauge Invariance Requirement. However, it is proved below that the SM literature ignores the Maxwellian Gauge Invariance Requirement. This issue emphasized the significance of this work.

Another important element of the QED Lagrangian density is that its interaction term is independent of derivatives of the quantum function $\psi_{,\mu}$. Hence, according to the result that is stated near the end of section 2, it does not modify the Noether expression for the 4-current (10).

3.2 Problems with Second-Order Quantum Equations

Let us carry out an analogous examination of second-order quantum equations of elementary massive charged particles. Here ϕ denotes the quantum function of a given particle. Specific theories of this kind are the mathematically complex KG equation and the electroweak equation of the $W^\pm$ particles. The Lagrangian density of the system $\mathcal{L}(\phi, \phi_{,\mu})$ depends on the quantum function ϕ and its derivatives. Therefore, by virtue of the Euler-Lagrange equation (5), a second-order quantum equation is obtained from a term that contains a product of derivatives of the particle's quantum function. In the case of the KG particle, one finds

$$\mathcal{L}_{KG} = a\phi^\dagger_{,\mu}\phi_{,\nu}g^{\mu\nu} + ..., \qquad (15)$$

where a is a numerical factor (see [4] p. 21; [7], p. 38; [13], p. 198). In the case of the electroweak theory, the quantum function is the 4-vector W^μ, and the Lagrangian density is

$$\mathcal{L}_W = a[\partial_\mu W_\nu - \partial_\nu W_\mu]^2 + ... \qquad (16)$$

(see [14], p. 307; [15], p. 518, [16], p. 113). Hereafter, χ denotes either the KG function ϕ or a component W_μ of the electroweak theory.

Since the dimension is a stiff property of a physical variable which does not vary continuously, one uses the previous results and derive these conclusions:

[label=CON.0] The $[L^{-4}]$ dimension of the Lagrangian density proves that the dimension of χ is $[L^{-1}]$. The 4-current is a real quantity whose dimension is $[L^{-3}]$. Therefore, the 4-current of a second-order charged particle should be a sum of terms of the form

$$j_\mu = a\chi^*_{,\mu}\chi + b\chi^*\chi_{,\mu}. \qquad (17)$$

Here the functions χ^*, χ are used because of the required mathematically real value of j^μ, and the derivative fixes its $[L^{-3}]$ dimension. The standard form of the electromagnetic interaction term of a charged quantum particle is

$$\mathcal{L}_{Int} = ej^\mu A_\mu. \qquad (18)$$

Relation (17) for the 4-current j^μ of the interaction term (18) contains a derivative of the quantum function χ. Therefore, due to the Noether theorem for the 4-current (10), one finds that this interaction term *yields a new term that belongs to the 4-current!* This new term contains the factors e and A_μ. For this reason, the new 4-current takes the form

$$j^\mu_{(new)} = j^\mu + e[\eta(A, \chi^*, \chi)]^\mu, \qquad (19)$$

where j^μ is the original 4-current and η is an appropriate function. The second term of the 4-current $j^\mu_{(new)}$ of (19) yields a new electromagnetic interaction term of the Lagrangian density

$$\mathcal{L}_{new} = eA_\mu j^\mu_{(new)} = e^2 A_\mu[\eta(A, \chi^*, \chi)]^\mu. \qquad (20)$$

Result (20) is totally unacceptable because this interaction term is not proportional to the electric charge e of the particle *and* it depends quadratically on the 4-potential A_μ. This outcome means that it violates the Maxwellian Gauge Invariance Requirement.

Conclusion A. Second-order theories of an elementary charged quantum particle are inherently wrong.

4 DISCUSSION

The totally uncorrectable result (20), that contains the second power of the 4-potential A_μ and the e^2 factor are found in the original publication of the KG theory (see eq. (37), p. 198 of [13]; see also [16], p. 73). Therefore, the KG theory of charged particles is wrong. These erroneous products are not found in quite a few textbooks on the electroweak $W^\pm$ particles simply because these textbooks adopt an unusual policy of refraining from a discussion of the electromagnetic interaction of these particles. However, ignoring a problem certainly does not mean that it is settled. And indeed, there are exceptional textbooks that show the electromagnetic interaction term of the $W^\pm$ Lagrangian density of the electroweak theory: The first term of the second line of eq. (C.18) on p. 518 of [15] shows a product of two A_μ as well as the erroneous factor e^2. An analogous expression is presented in eq. (11.31), on p. 113 of [16]. It means that also the electroweak theory of the $W^\pm$ is wrong. Hence, there are inherent contradictions in second-order quantum theories of an elementary charged particle.

The following information provides strong support for the validity of the contradictions that are derived above, namely, for conclusion A which is shown near the end of the previous section. It means that the SM electroweak theory of the $W^\pm$ particles is wrong because it lacks a coherent expression for the electromagnetic interaction of the electrically charged $W^\pm$ particles.

- The Dirac theory of the electron was published in 1928 [17]. About one month later Darwin published a coherent expression for the 4-current of the Dirac particle [18]. This example shows that in the case of a consistent theory, the required 4-current is found right away.
- The fate of the 4-current of the electroweak theory of the $W^\pm$ is completely different. The electroweak theory was created in the 1960s. About 20 years later several authors published an *effective* expression for the $W^\pm$ electromagnetic interactions [19]. Their expression is still used by thousands of people who work in very large research centers such as Fermilab and CERN [21, 20]. Their effective interaction term depends on derivatives of the W function (see eq. (2.1) of [19]). Items 1 - 5 of the previous section prove that an application of a 4-current of an electrically charged particle that contains a derivative of the quantum function is a gross theoretical error.
- In contrast to the case of the Dirac particle of the first item of this list, where the 4-current was found after one month, more than 50 years have elapsed since the publication of the electroweak theory, but there is still no coherent theoretical expression for the $W^\pm$ 4-current that is required for its electromagnetic interaction.

These points strongly indicate that it is really impossible to describe consistently the electromagnetic interactions of the electroweak theory of the $W^\pm$ particles.

5 RELEVANT PROBLEMS

It is proved above that dimension problems emerge from the Lagrangian density of second-order quantum theories. Incoherent expressions for electromagnetic interactions demonstrate these problems. However, a rule of thumb says that it is very likely that an erroneous structure of a theory should yield more than one specific error. This section supports this opinion. It discusses another aspect of the erroneous structure of second-order quantum theories. A typical attribute of these theories is the mass term of their Lagrangian density whose form is

$$\mathcal{L}_{Mass} = -bm^2\chi^*\chi, \qquad (21)$$

where m is the particle's mass and b is a numerical constant. The product $\chi^*\chi$ is a mathematically real expression whose dimension is $[L^{-2}]$. Therefore, the $[L^{-4}]$ dimension of the

Lagrangian density yield the second power of the mass of (21)

Let us see how this term affects the structure of the energy-momentum tensor $T^{\mu\nu}$ of second-order theories. One example of the importance of this tensor is that $T^{\mu 0}$ is the energy-momentum density of the system (see [9], pp. 84, 85; [6] p. 19). Hence, the spatial integrals of $T^{\mu 0}$ are the system's energy-momentum. Energy-momentum are quantities that are used in quantum mechanics and classical physics. For example, the physical energy-momentum are used in the de Broglie expression for the phase of a free particle

$$\Phi \propto e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}. \tag{22}$$

Here, $\boldsymbol{k}$ is the particle's momentum $\boldsymbol{p}$, and ω is its energy (see [1], pp. 3, 18; [3], pp. 119, 120). Therefore, the generalized correspondence principle says that every acceptable QFT should provide a consistent definition of the energy-momentum tensor.

Another example of the need for this tensor is the Einstein equation for the gravitational fields (see [9], p. 297)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi k T_{\mu\nu}. \tag{23}$$

Here $R_{\mu\nu}$ is the curvature tensor and k is the gravitational constant (see [9], p. 288). The ordinary energy-momentum tensor $T_{\mu\nu}$ that is obtained for a flat space-time coordinates where the metric is uniform, agrees with the general relativistic expression (see [9], p. 292). Let us utilize the Noether theorem that shows how the required energy-momentum tensor is derived from the theory's Lagrangian density

$$T^{\mu\nu} = g^{\mu\alpha}\psi_{,\alpha}\frac{\partial\mathcal{L}}{\partial\psi_{,\nu}} - \mathcal{L}g^{\mu\nu} \tag{24}$$

(see [4], p. 311; [6], p. 19).

The discussion analyzes the theory's mass term, and it follows the order of section 3. It begins with an examination of the first-order Dirac theory of a quantum function of a massive particle that is used in the QED Lagrangian density (6), and it shows the coherent interrelations between theoretical elements. Later, the analysis proves that unsettled problems exist with second-order quantum theories.

5.1 The First-Order Dirac Theory of a Quantum Function

The Noether expression for the energy-momentum tensor $T_{\mu\nu}$ of (24) changes the form of terms that depend on derivatives of the quantum functions $\psi_{,\nu}$. On the other hand, derivative-independent terms just change sign and are multiplied by the metric tensor $g^{\mu\nu}$. The mass term of the QED Lagrangian density (6) is derivative-free, and it yields this term of the corresponding energy-momentum tensor

$$T^{\mu\nu}_{QED} = \bar{\psi}m\psi g^{\mu\nu} + ... \tag{25}$$

This term is consistent with the mass term of the standard form of the energy-momentum tensor (see [9], p. 92). In particular, the component T^{00} is the energy density of the system (see [6], p. 19; [9], p. 92). This analysis shows that the mass term of the Dirac QED equation (25) is a mathematically positive expression that agrees with the linear dependence of energy on mass. Therefore, the Dirac linear equation of a spin-1/2 massive particle agrees with the required form of the energy-momentum tensor.

5.2 Second-Order Theories of a Quantum Function

It is already stated above that the mass term of the Lagrangian density of a second-order theory of a quantum function (21) takes the form $am^2\phi^\dagger\phi$. Here a is a numerical constant. Hence, the energy-momentum tensor of a second-order theory of a quantum function is

$$T^{\mu\nu}_{2ND} = bm^2 g^{\mu\nu}F(\chi^\dagger, \chi) + ..., \tag{26}$$

where $F(\chi^\dagger, \chi)$ is a scalar expression that depends on the quantum functions. This quadratic mass expression is totally unacceptable because it violates the linear dependence of energy on mass.

It should be noted that the Higgs theory goes from bad to worse. Indeed, the mass term of the Higgs Lagrangian density is a positive expression

$$\mathcal{L}_{Higgs} = m^2\phi^*\phi + ... \tag{27}$$

(see [6], p. 715; [15], p. 515; [22]). Hence, the Higgs energy-momentum tensor is

$$T^{\mu\nu}_{Higgs} = -m^2 g^{\mu\nu}\phi^*\phi + ... \qquad (28)$$

It means that for either a positive or negative Higgs mass $\pm m$, the energy density T^{00} of the Higgs theory is negative. In other words:

The Higgs theory says that Nature contains a particle whose energy density is negative and its entire energy is negative!

This result is totally inconsistent with experiment. In particular, it denies the claim that the 125 GeV particle is a Higgs boson [11].

6 CONCLUDING REMARKS

The unit system used herein reduces the number of dimensions of a physical quantity to unity and it takes the form $[L^n]$. Furthermore, the dimension of a physical quantity takes a unique value throughout every process. Therefore, the dimensional analysis that is carried out above is quite simple, and the correctness of the mathematical arguments is beyond doubt.

This work regards the QED Lagrangian density (6) as the fundamental expression of the quantum description of an interacting system that comprises elementary massive charged particles and electromagnetic fields. This expression relies on the first-order Dirac equation of a charged spin-1/2 massive particle. As stated in section 2, experiments strongly support the QED Lagrangian density (6). The specific analysis of this work supports the theoretical structure of QED.

An examination of electromagnetic interactions yields two quite new theoretical elements that are explained and utilized above:

- The quantum function of a Lagrangian density has dimensions. The dimension of the quantum function ψ of the Dirac linear theory is $[L^{-3/2}]$. This dimension yields the coherent expressions of QED. In contrast, second-order quantum theories use a function ϕ whose dimension is $[L^{-1}]$. Unsettled problems emerge from this dimension. For example:

  A coherent 4-current cannot be constructed for second-order quantum theories. Hence, *the ordinary QED Lagrangian density (6) of a charged Dirac particle and electromagnetic fields cannot be extended to include second-order quantum theories.*

- Terms of a Lagrangian density that depend on a power $n > 1$ of the electromagnetic 4-potential A_μ are unacceptable because they violate the gauge invariance of electrodynamics. One reason proving that second-order quantum theories are wrong is that they have terms of this kind. This restriction is still not well-known and some textbooks explicitly disobey it. It follows that the publication of this work is badly needed.

Section 5 examines the energy-momentum tensor of quantum theories. It proves that besides the analysis of electromagnetic interactions, this tensor supports the QED Dirac equation. On the other hand, this section proves inherent contradictions of the energy-momentum tensor of second-order quantum theories. In particular, the Higgs theory yields the totally unacceptable result of an energy-momentum tensor whose energy density and its total energy are negative!

It is interesting to remark that the conclusions of this work agree with Dirac's lifelong objection to second-order quantum theories (see [23], pp. 1-8).

COMPETING INTERESTS

Author has declared that no competing interests exist.

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