Modeling of a Condenser in Dynamic Operation Using Comsol Multiphysics Software

Jean-Louis Comlan Fannou¹,²,³*, Kouandété Valéry Doko¹, Vincent Prodjinonto¹, Louis Lamarche², Stanislaw Kaji², Gérard Degan¹ and Emile A. Sanya¹

¹Laboratoire d’Energétique et de Mécanique Appliquées (LEMA), École Polytechnique d’Abomey-Calavi, 01 BP 2009 Cotonou, Benin.
²Thermal Technology Center (TTC), Department of Mechanical Engineering, École de Technologie Supérieure, Université du Québec, 1100, Notre-Dame Street West, Montréal, H3C 1K3, Canada.
³Ecole Nationale Supérieure de Génie Énergétique et Procédés (ENSGEP)/UNSTIM BP 2282 GOHO Abomey, Benin.

Authors’ contributions

This work was carried out in collaboration among all authors. Author JLCF designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors KVD, VP, LL and SK managed the analyses of the study. Authors GD and EAS managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/PSIJ/2020/v24i830206

Editors:
(1) Dr. Olalekan David Adeniyi, Federal University of Technology, Nigeria
(2) Dr. Thomas F. George, University of Missouri-St. Louis, USA

Reviewers:
(1) Sandeep Bhardwaj, CCS Haryana Agriculture University, India
(2) Ratnakar Ramchandra Kulkarni, Universiti Malaysia Pahang, Malaysia
(3) Abayomi Ayotunde Ayoade, University of Lagos, Nigeria
(4) Yusuf Oloruntoyin Ayip, Kwara State University, Nigeria

Complete Peer review History: http://www.sdiarticle4.com/review-history/61067

Received 15 July 2020
Accepted 19 September 2020
Published 12 October 2020

ABSTRACT

This paper is about modelling in dynamic operation the heat exchanger system using as condenser in one dimension characterized by two coaxial tubes with ribbed inner tube by using the equations of mass, momentum and energy conservation. The Comsol PDE interface is used to simulate the monophasic and biphasic flows of refrigerant. Heat transfer in water and inner wall of the condenser are modeled with two Heat Transfer Interfaces (solid, fluid) in Comsol software. The model has been validated by comparing the numerical and experimental results obtained with the direct expansion geothermal heat pump. The analysis of the comparative results shows that the

*Corresponding author: E-mail: jlfannou@gmail.com;
obtained model adequately fits the experimental data with an average deviation of less than 5%. Therefore, it proves that it’s a good model which can be used for simulation purposes. This developed numerical model was used to simulate the superheating, condensing and subcooling phases in the condenser. Vapor quality, pressure, enthalpy of the refrigerant and water temperature are also simulated.

**Keywords:** Dynamic operation; comsol multiphysics software; deviation; PDE.

**ABBREVIATIONS**

- \( p \) : Pitch of the ribs (m)
- \( e \) : Rib height (m)
- \( \alpha \) : Helix angle of the ribs (°)
- \( N \) : Number of departures of ribs per section
- \( s \) : Length of the base of the ribs (m)
- \( D_{e} \) : Maximum internal diameter (m)
- \( t \) : Wall thickness of the tube twisted (m)
- \( D_{b} \) : Minimum inner diameter (m)
- \( D_{1p} \) : Equivalent inside diameter of the tube means within the perimeter (m)
- \( D_{1A} \) : Inside diameter of the tube equivalent procedure on the basis of average (m)
- \( D_{2A} \) : Outer diameter of the tube equivalent procedure on the basis of average (m)
- \( D_{2p} \) : Outer diameter of the tube equivalent means within the perimeter (m)
- \( Re \) : Reynolds number
- \( \mu \) : Viscosity of refrigerant (N.s/m²)
- \( D \) : Annular diameter (m)
- \( r^* \) : Ratio of radii for the ring
- \( e^* \) : Dimensionless rib height
- \( \theta^* \) : Dimensionless helix angle
- \( p^* \) : Dimensionless pitch of the ribs
- \( \rho \) : Density of refrigerant (kg/m³)
- \( m \) : Refrigerant mass flow (kg/s)
- \( S_{f} \) : Section fluid passage (m²)
- \( P \) : Refrigerant pressure (Pa)
- \( h \) : Refrigerant enthalpy (J/kg)
- \( Q_{v} \) : Heat flux per unit exchanged with the inner wall (W/m³)
- \( F_{vol} \) : Frictional force per unit volume between the tubes and refrigerant (N/m³)
- \( \chi \) : Vapor quality
- \( \rho_{p} \) : Density of the inner wall (kg/m³)
- \( A_{p} \) : Axial wall area (m²)
- \( C_{p} \) : Specific heat of the wall (J/kg.K)
- \( k_{p} \) : Thermal conductivity of the wall (W/m.K)
- \( h_{rp} \) : Coefficient of heat exchange between the refrigerant and the wall (W/m².K)
- \( h_{wp} \) : Coefficient of heat exchange between the inner wall and water (W/m².K)
- \( h_{rpmono} \) : Coefficient of exchange between the refrigerant and the wall in monophasic flow (W/m².K)
- \( H_{rpmono} \) : Coefficient of exchange between the refrigerant and the wall in two-phase flow (W/m².K)
- \( H_{totC} \) : The overall coefficient of heat exchange simultaneously taking into account the single-phase and two-phase flows of the refrigerant (W/m².K)
- \( h_{f} \) : Enthalpy of refrigerant in liquid phase depending on the pressure (kJ/kg)
- \( h_{g} \) : Enthalpy in gas phase refrigerant depending on the pressure (kJ/kg)
1. INTRODUCTION

This research study is part of a large project to improve and develop the direct expansion (DX) geothermal heat pump and to evaluate the performance of such a system with potential fluids (R404A and R407C) for the substitution of refrigerant R22 which, as expected, must disappear because of its polluting nature [1]. But before moving to R410A and R407C, it is advisable to model the DX system for the R22 in order to have reference data, which justifies this study. A direct expansion geothermal heat pump (DX GHP) is a system having the particularity that the geothermal heat exchanger buried in the ground is a component of the heat pump which can play the role of condenser / evaporator depending on the operating mode (Fig. 1) contrary to the traditional secondary loop geothermal heat pump. Thus, it has the following advantages among others [2]:

- Reduction of cost by the elimination of secondary loop of the ground side.
- Low power consumption for its operation.
- Using a larger energy resource with a relatively constant temperature throughout the year.
- Good even at very low atmospheric temperature.
- Reduced maintenance costs.

As main drawback, the initial investment costs are very high, but because of the energy savings, return on investment can be done quickly. That is why geothermal systems have recently renewed interest in recent years [3].

\[ T_p \] : Temperature of the inner wall (K).
\[ \text{Pr} \] : Prandtl number
\[ \text{Nu} \] : Nusselt number
\[ F_r \] : Froude number
\[ u \] : Refrigerant velocity (m/s)
\[ W_e \] : Weber number
\[ k_r \] : Thermal conductivity of refrigerant (W/m.K)
\[ k_e \] : Thermal conductivity of water (W/m.K)
\[ k_f \] : Thermal conductivity of the liquid phase (W/m.K)
\[ L \] : Condenser length (m)
\[ C_{p_f} \] : Specific heat of the liquid phase (J/kg.K)
\[ D_3 \] : Inner diameter of the outer tube (m)
\[ \text{PDE} \] : Partial Differential Equations
\[ D_1 \] : Inner diameter of the inside tube (m).
\[ D_2 \] : Outside diameter of the inside tube (m).
\[ h_{in} \] : Input Enthalpy of refrigerant (kJ/kg)
\[ h_{out} \] : Output Enthalpy of refrigerant (kJ/kg)
\[ p_{in} \] : Input refrigerant pressure (Pa)
\[ p_{out} \] : Output refrigerant pressure (Pa)
\[ T_w \] : Water temperature in the condenser (K).
\[ T_{win} \] : Input water temperature in the condenser (K).
\[ T_{wout} \] : Output water temperature in the condenser (K).
\[ \dot{m}_w \] : Water flow rate in the condenser (kg/s)
\[ \dot{m}_{out} \] : Output refrigerant mass flow (kg/s)
\[ \rho_{in} \] : Input refrigerant density (kg/m$^3$)
\[ \rho_w \] : Water density (kg/m$^3$)
\[ V_{ites} \] : Velocity of water (m/s);
\[ C_w \] : Specific heat of water (J/kg.K);
\[ \nu_w \] : Velocity of water (m/s);
\[ Q_w \] : heat exchange between the water and the wall
\[ Q_P \] : heat exchange between the wall with water and refrigerant
\[ C_f \] : Specific heat of refrigerant (J/kg.K);
\[ T_f \] : Temperature of refrigerant (K)
The literature review revealed a lack of scientific research and publication concerning direct expansion geothermal heat pump systems but in recent years studies on DX systems have been published and listed by Fannou et al [2]. In terms of modeling and experimental results, the existing works in the field do not inform enough about this technology. Therefore, the proposed modeling and analysis of this heat pump DX aims to fill this gap.

Modeling and analysis of a direct expansion geothermal heat pump begins with modeling these different components: ground heat exchanger, compressor, thermostatic expansion valve, reversing valve, pipe, water-refrigerant exchanger, etc. and the coupling of these components to make a closed loop corresponding to the heat pump.

In the first part, the numerical model of ground heat exchanger had been already described, simulated, experimentally validated and presented by our team [4].

In this study, a dynamic model of the second heat exchanger system used as a condenser which was the subject of a conference in Boston in 2012 was presented[5]. The results have been improved and the model has been refined.

The model presented is based on solving the equations from mass, momentum and energy conservations. The obtained numerical model was used to simulate the different phase changes as superheating, condensing and subcooling. The variation of vapor quality as well as the temperature, the pressure of the refrigerant along the condenser are also presented. The resulting model was then validated by comparison with experimental results.

In the literature, some models have been developed. For example, Ndiaye [6] proposed a model which served as a basis to develop the initial model. Difference with our new model is that the experimental data are performed on a secondary loop geothermal heat pump with lower discharge pressures than on our DX device where discharge pressures and pressure drops much higher. Moreover, in order to validate our obtained new model, the Zvi vacuum rate correlation has been replaced by that of Prémoli [7] and an adjustment of the heat transfer correlations has also been made.

The implementation of this approach in Comsol multiphysics software is for an educational purpose in that it shows an example of modeling and coupling tips that are not necessarily well-known. The main objective of this study is therefore to set up a dynamic model of a condenser available to the scientific community and, most importantly, to demonstrate how to use modeling and coupling tips with Comsol software.

![Fig. 1. Simplified diagram of a direct expansion geothermal heat pump](image-url)
It should be emphasized that the model presented in this study was coupled in a global model of the direct expansion geothermal heat pump [4] entirely developed under Comsol software.

2. MATERIALS AND METHODS

2.1 Geometry Description

The coaxial heat exchanger spirally wound tube whose inner tube is called twisted or ribs is modeled (Fig. 2). There is characterized the point of view of internal geometry by: p, e, α, N, De, t, Db (Fig. 3).

The advantage of such geometry is to increase the exchange surface area to maximize heat transfer within it. Because of this irregular geometry in inner tube, the authors [8, 9] proposed the equivalent of a corresponding smooth tube. The equivalent diameters are calculated using the following formulas:

\[ D_{1A} = \sqrt{D_e^2 - \frac{2Ne_s}{\pi}} \]  
\[ D_{1P} = D_e + \frac{N}{\pi} \left( \sqrt{4e^2 + s^2} - s \right) \]  
\[ D_2 = D_1 - 2t \]
According to Rousseau et al. [9], the ribs helix angle can be calculated by:

\[
\alpha = \arctan \left( \frac{\pi D_2 A}{Np} \right)
\]  

(4)

2.2 Modeling using Comsol Multiphysics Software

The condenser is installed on a direct expansion geothermal vapor compression heat pump with reversible cycle power of 10 kW. The refrigerant circulates in the annular space and the secondary fluid (water) into the inner tube (Fig. 4). Refrigerant (R22) may flow as monophasic or biphasic. The outer wall of the condenser is considered to be isolated and that there is therefore, there is no heat transfer between the refrigerant and the outer wall. The temperature of the refrigerant is then equal to that of the outer wall at every point. The modeling presented here, describes three elements that interact: refrigerant, secondary fluid (water) and the inner tube wall. The following hypotheses have been formulated in order to write the governing equations: the refrigerant is assumed Newtonian, the flow is one dimensional along the axis of the tube, the gravitational force is negligible, the axial conduction is negligible, and the viscous dispersion is negligible.

Considering the set of equations to solve, modeling in Comsol is based on the choice of three modules: the PDE module coupled with two heat transfer modules (solid, liquid) are used to solve our equations. The heat transfer module liquid is used to solve the equations of water flow, while the solid heat transfer interface is used to solve the heat transfer within the tube inside the exchanger. The input variables of the model are: enthalpy, pressure, density, refrigerant flow rate, inlet temperature and the cooling water flow rate. The followings stand as output variables: enthalpy, pressure and flow rate of the refrigerant, outlet temperature of the cooling water, vapor quality (Fig. 5).

![Fig. 4. A schematic sectional view of the studied condenser](image1)

![Fig. 5. Inputs and outputs variables of the condenser model](image2)
2.2.1 The refrigerant

Based on the governing equations published in Fannou et al.[5] and Ndiaye et al.[10], (see appendix), the single-phase and two-phase refrigerant flows are modeled with the PDE module of Comsol software that occurs in annular space of the condenser.

Equations A-1 to A-29 can be written in general form according to the Comsol model.

\[
e^a_u \frac{\partial^2 u}{\partial t^2} + a^a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \nabla u + \alpha u = f
\]

\( u = [\dot{m}, \rho, p, h]^T \), is the single column matrix of variables

\( \nabla = \left[ \frac{\partial}{\partial x} \right] \), represents the partial derivative operator.

- The diffusion coefficient \( c \) in actual fact is represented by the null matrix.
- The absorption coefficient \( a \) is represented by the matrix below:

\[
  a = \begin{bmatrix}
    0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
  \end{bmatrix}
\]

- The generalized source term \( f \) is modeled by the one-column matrix below:

\[
  f = \begin{bmatrix}
    0 \\
    -S^2_f * F_{tot}(h, \dot{m}, P) \\
    Q_{totc}(h, \dot{m}, T_p, P) \\
  \end{bmatrix}
\]

- The mass coefficient \( e_a \) in our case this study is a null matrix
- The Damping or Mass Coefficient \( d_a \) is modeling as:

\[
  d_a = \begin{bmatrix}
    0 & S_f & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    S_f & 0 & 0 & 0 \\
    0 & 0 & -S_f & S_f * \rho \\
  \end{bmatrix}
\]

- The Conservative Flux Convection Coefficient \( \alpha \) is null
- The Convection Coefficient \( \beta \) is defined by:

\[
  \beta = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
    2*S_f^2 / \rho^2 & S_f^2 & 0 \\
    0 & 0 & -S_f / \rho & \dot{m} \\
  \end{bmatrix}
\]

- The Conservative Flux Source \( \gamma \) is null

In these relations, \( V_{tes} \) and \( Q_{totc} \), \( F_{tot} \) respectively represent the refrigerant flow velocity, the exchanged heat and the frictional pressure drop. They are modeled in Comsol software as a functions depending on their variables by taking into account the single-phase and two-phase flows of the refrigerant.

For example, \( Q_{totc} \) which characterizes the heat exchanged between the internal wall and the refrigerant is written in Comsol as:

\[
  Q_{totc} = alfa * coef * H_{totc}(h, V_{tes}(h, \dot{m}, P), T_p, P) * (T_p - T_f(h, P))
\]
\( \alpha f a = 1.13, \) is obtained by adjustment with experimental data

With:

\[
coef = \frac{4 \cdot D_p}{D^2 - D^2_a} \tag{10}
\]

Coef is used as a parameter in Comsol software

\[
H_{totC} = h_{tropmo} (h, u, P) \cdot \left( f l c 2 h s (h_f (P) - h, 0.01) \right) + H_{tpd} (h, u, T_p, P) \cdot \left( f l c 2 h s (h - h_f (P), 0.01) \right) + (f l c 2 h s (h - h_f (P), 0.01)) \cdot \left( f l c 2 h s (h_g (P) - h, 0.01) \right) \tag{11}
\]

Flc2hs is a predefined function in Comsol that allows to couple the two refrigerant flow phases in the condenser.

### 2.2.2 The secondary fluid (water)

It can be assumed that the incompressible fluid flow is unidirectional while the axial conduction and viscous dissipation are neglected. Therefore, the energy conservation equation of energy applied to the water and it is modeled by using the module 'Liquid Heat Transfer Module', the general equation of which reads as follows:

\[
\rho_w C_w \frac{dT_w}{dt} + \rho_w C_w v_w \nabla T_w = Q_w \tag{12}
\]

\[
Q_w = \frac{4 \cdot D^2_p}{D^2_a} h_{cp} \cdot \left( T_p - T_w \right) \tag{13}
\]

\( h_{cp} \) is the heat exchange coefficient between the water and the wall and is a function of the flow velocity of the water (Appendix A-30 to A-34).

### 2.2.3 The inner wall

The equation of energy conservation applied to the inner wall can be written assuming a constant thermal conductivity and is modeled using the module 'Solid Heat Transfer Module' whose general equation is presented as follows:

\[
\rho_p C_p \frac{dT}{dt} = \nabla \cdot \left( k_p \nabla T \right) + Q_p \tag{14}
\]

With:

\[
Q_p = \left[ -p_i \cdot D_{1p} \cdot h_{ep} (v_w) \cdot \left( T_p - T_e \right) + p_i \cdot D_{2p} \cdot H_{totC} (h, \text{Visc}(h, m, P), T_p, P) \cdot \left( T_f (h, P) - T_p \right) \right] / A_p \tag{15}
\]

\( p_i = \pi, \) is used as a parameter in Comsol software.

### 2.2.4 Meshing and boundary conditions

Since this is a dynamic model of the condenser that it has been developed, it is important to specify the conditions for entering the refrigerant into the heat exchanger. In agreement with Fig. 5, the following boundary and initial conditions have been considered.

*Water:* Temperature and flow are known at the input \((z = 0).\)
Refrigerant: Temperature, pressure, flow rate and vapor quality are known at the input level (z = 0).

Inner tube: Heat flux is zero at the boundary.

We have assumed a one-dimensional model. Thus, the mesh chosen in Comsol is the "fine". An extract of the one-dimensional mesh obtained is presented in Fig. 6. The number of nodes is 251 for a mesh length of 6.3 m with a maximum growth rate of the elements of 1.3. The maximum size of the elements is 0.334 m with a thin region resolution of 1.

Fig. 6. Domain meshing

2.2.5 The refrigerant properties

In most cases, it is calculated the refrigerant properties using the direct interface between the Comsol software and REFPROP software via MATLAB environment but by doing so, the calculation time is very long based on our experience in the field. In our case, a MATLAB script via the REFPROP software that provides an array of refrigerant properties have been developed. This data table is entered into Comsol and by interpolation, Comsol calculates the refrigerant properties. The simulation time is then very greatly improved.

3. RESULTS

3.1 Model Validation

The condenser modeled and presented in this study has been installed on an experimental device at the Thermal Technology Center of the High Technology School in Montreal, Canada. It is a direct expansion geothermal heat pump consisting of three geothermal loops of 30 m each, installed in parallel and can operate in heating mode [2] as well as in cooling mode [11]. This reversible heat pump has a nominal cooling capacity of 10 kW and uses the R-22 (chlorodifluoromethane) fluid as a refrigerant. The device has a heat exchanger of Turbotec BTSSC-60 type which acts as condenser or evaporator according to the operation mode. The thermocouples are installed in the loops at different positions as shown in Fig. 7. The AC compressor of Tecumseh AVA5538EXN type has a rated power of 2.24 kW. The expansion valves used differ according to the system operation mode. In cooling mode, it is a single expansion valve named TXV cooling (Fig. 7) (model Parker SE5VX100) with a 17.0 kW nominal capacity is used, while in heating mode, there are three expansion valves named TXV heating (Fig. 7) (model Danfoss TUBE068U2162) with a 2.6 kW nominal capacity each are used.

The condenser was simulated for 20 min and the results were compared with those obtained experimentally in the same simulation conditions. It should be noted that the tests were repeated to ensure repeatability and reproducibility in the measurements. Table 1 shows the condenser's parameters and the Figs. 7 to Fig. 9 show the obtained results.

Fig. 7 shows the results of the validation of the cooling water's temperature in condenser as compared to the experimental results. As you can see, the experimental results fit the model well. The mean difference observed is –4% compared to the experimental values.

Fig. 8 shows the results of the refrigerant pressure's validation at the condenser outlet compared to the experimental results. As can be seen, the experimental results match the model with a mean difference about of 2% compared to the experimental values.

The Fig. 9 depicts the results of the refrigerant enthalpy's validation at the
condenser outlet compared with the experimental results. As can be seen, the experimental data fit well to those from the obtained model. The mean difference observed is about 2.4% compared to the experimental.

![Experimental diagram in heating mode](image)

**Fig. 7. Experimental diagram in heating mode**

**Table 1. Condenser’s parameters**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>m</td>
<td>6.30</td>
</tr>
<tr>
<td>D&lt;sub&gt;e&lt;/sub&gt;</td>
<td>mm</td>
<td>33.75</td>
</tr>
<tr>
<td>D&lt;sub&gt;1A&lt;/sub&gt;</td>
<td>mm</td>
<td>25.00</td>
</tr>
<tr>
<td>D&lt;sub&gt;1P&lt;/sub&gt;</td>
<td>mm</td>
<td>51.61</td>
</tr>
<tr>
<td>D&lt;sub&gt;2A&lt;/sub&gt;</td>
<td>mm</td>
<td>34.33</td>
</tr>
<tr>
<td>D&lt;sub&gt;2P&lt;/sub&gt;</td>
<td>mm</td>
<td>53.80</td>
</tr>
<tr>
<td>D&lt;sub&gt;3&lt;/sub&gt;</td>
<td>mm</td>
<td>38.51</td>
</tr>
<tr>
<td>e</td>
<td>mm</td>
<td>4.20</td>
</tr>
<tr>
<td>p</td>
<td>mm</td>
<td>13.51</td>
</tr>
<tr>
<td>θ&lt;sup&gt;*&lt;/sup&gt;</td>
<td>C</td>
<td>0.75</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Kg/m&lt;sup&gt;3&lt;/sup&gt;</td>
<td>8300</td>
</tr>
<tr>
<td>k&lt;sub&gt;p&lt;/sub&gt;</td>
<td>W/m.K</td>
<td>419</td>
</tr>
<tr>
<td>C&lt;sub&gt;f&lt;/sub&gt;</td>
<td>J/Kg.K</td>
<td>372</td>
</tr>
</tbody>
</table>
Fig. 8. Cooling water temperature validation

Fig. 9. Refrigerant pressure validation
3.2 Thermodynamic Parameters Variation along the Condenser

The validated model is used to simulate the thermodynamic parameters along the condenser. As can be seen in each of the figures (for example in Fig. 11), the three phases of condensation are observed: the superheating (Phase I), the condensation (Phase II) and the subcooling (phase III).
Fig. 52. Refrigerant enthalpy variation

Fig. 63. Refrigerant pressure variation
4. CONCLUSION

The aim of this work was to provide the scientific community with a dynamic model of a condenser and to show how to implement this model in Comsol software. To reach that objective, the numerical model of the condenser by solving the equations of mass, momentum, and energy conservation, while taking into account thermodynamic properties, in one-dimension, have been implemented and validated experimentally. The average of the differences between the results of numerical modeling and those from experiment is less than 5%. This justifies the quality and performance of the model presented so far. It would be recommend to model the condenser in two dimensions even three dimensions for purposes of analysis and comparison. The current project is to be presented in our coming publications in relation with a condenser modelled with R22 replacement refrigerants such as R407C, R410A.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

APPENDIX: GOVERNING EQUATION MODELING THE CONDENSER

Fannou et al.[5] et Ndiaye et al. ([10]).

Monophasic flow

According to the assumptions cited previously, the governing equations are:

\[ S \frac{\partial \rho}{\partial t} + \frac{\partial \dot{m}}{\partial z} = 0 \quad (A-1) \]

\[ S \frac{\partial \dot{m}}{\partial t} + \frac{2 \dot{m} \vec{m} - m^2 \frac{\partial \rho}{\partial z}}{\dot{\rho}} - S \frac{\partial P}{\partial z} = -S F_{vol} \quad (A-2) \]

\[ S f \frac{\partial h}{\partial t} + \frac{\dot{m} \vec{h}}{\partial z} - S f \frac{\partial P}{\partial z} = S f Q_v \quad (A-3) \]

with:

\[ F_{vol} = 2 C \int \dot{m}^2 / S f \int \frac{\rho}{D} \quad (A-4) \]

Where \( C_f \) represents the friction coefficient. The friction coefficient is given by the Garimela correlation [12]:

For \( Re < 800 \),

\[ C_f = \frac{24 r^{*0.035}}{Re} \left( 1 + 101.7 Re^{0.55} \times \exp^{*1.65 + 2.0l} \times r^{*5.77} \right) \quad (A-5) \]
For $\text{Re} \geq 800$,

$$
C_f = \left[ 1.7372 \ln \left( \frac{\text{Re}}{1.964 \ln \text{Re} - 3.8215} \right) \right]^{-2} (A-6)
$$

$$
e_f = \left( 1 + 0.0925 r^* \right) e_f
$$

$$
\left( 1 + 222 \text{Re}^{0.09} e^{*2.40} p^{*0.49} \theta^{*-0.38} r^{*2.22} \right) (A-7)
$$

In these equations,

$$
\text{Re} = \rho uD / \mu_f (A-8)
$$

$$
D = D_3 - D_{2A} (A-9)
$$

$$
r^* = D_{2A} / D_3 (A-10)
$$

$$
e^* = e / D_{2A} (A-11)
$$

$$
\theta^* = \alpha / 90 (A-12)
$$

$$
p^* = p / D_{2A} (A-13)
$$

The heat exchanged with the inner tube is given by the relation:

$$
Q_v = \frac{4}{D_3^2} \frac{D_{2p}}{D_{2A}^2} \left( T_p - T_f \right) (A-14)
$$

Heat transfer coefficient between wall and refrigerant is calculated by Garimela correlation [12]:

$$
h_{wp} = \frac{k_f}{D} \left[ \frac{0.5C_f \text{Re Pr}}{1 + 9.77 \sqrt{0.5C_f \left( \text{Pr}^{2/3} - 1 \right)}} \right] \left( \text{Re}^{-0.20} e^{*-0.32} p^{*-0.28} r^{*-1.64} \right) (A-15)
$$

**Two-phase flow**

It is assumed that the flow is homogeneous and that the two phases are in thermodynamic equilibrium. The governing equations take the same form as in single-phase flow except that the parameters that apply to the liquid-vapor mixture such as follows:

The density as a function of the void fraction:

$$
\rho = \left[ 1 - \alpha_b \right] \rho_f + \alpha_b \rho_g (A-16)
$$

$f$ index refers to the liquid phase and the index $g$ refers to the vapor phase.

The enthalpy can be written as:
In this model, the void fraction $\alpha_b$ proposed by Prémoli [7] is used.

The frictional pressure drop is evaluated assuming a single phase flow corrected with two-phase multiplier [6]:

$$ (F_{vol})_{di} = \phi^2 (F_{vol})_{mono} $$  \hspace{1cm} (A-18)

mono index refers to monophasic flow and di index refers to biphasic flow. $\phi^2$: The biphasic multiplier factor.

$$ \phi^2 = A_1 + \frac{3.24 A_2 A_3}{Fr^{0.045} We^{0.035}} $$  \hspace{1cm} (A-19)

Where:

$$ A_1 = (1-x)^2 + x^2 (\rho_f f_g / \rho_v f_f) $$  \hspace{1cm} (A-20)

$f_g = C_f$: For the gaseous phase flow

$f_f = C_f$: For the liquid phase flow

$$ A_2 = x^{0.78} (1-x)^{0.224} $$  \hspace{1cm} (A-21)

$$ A_3 = (\rho_f / \rho_g)^{0.69} (\mu_g / \mu_f)^{0.19} (1-\mu_g / \mu_f)^{0.7} $$  \hspace{1cm} (A-22)

$$ Fr = \frac{u^2}{gD} $$  \hspace{1cm} (A-23)

$$ We = \frac{\rho u^2 D}{\sigma} $$  \hspace{1cm} (A-24)

$\sigma$, Stefan's constant.

The heat transfer coefficient in condenser of wall-refrigerant is: Koyama’s Correlation[13]

$$ h_{wp} = 0.53 \left( \frac{k_f}{d} \right)^{-0.4} \left( \frac{d}{D} \right)^{0.4} Ph^{-0.6} \left( \frac{Re_u Pr_f}{R} \right)^{0.8} $$  \hspace{1cm} (A-25)

with:

$$ Ph = Cp_f (T_r - T_p) / (h_g - h_f) $$  \hspace{1cm} (A-26)

$$ Re_u = \frac{\rho_f u d}{\mu_f} $$  \hspace{1cm} (A-27)

$$ Pr_f = \frac{\mu_f Cp_f}{k_f} $$  \hspace{1cm} (A-28)
The secondary fluid (water)

It is assumed that the incompressible fluid flow is unidirectional. In addition, it is neglected axial conduction and viscous dissipation. The equation of energy conservation in the secondary fluid can be written:

\[ \rho_e C_p \left( \frac{\partial T_e}{\partial t} + u \frac{\partial T_e}{\partial z} \right) = \frac{4D_p h_{sp}^2}{D_i^2} \left(T_p - T_e\right) \]  

(A-30)

Expression of \( h_{sp} \)

The correlation proposed by Ravigurajan and Bergle [14] that take into account the internal geometry of the inner tube is used:

\[ h_{sp} = \frac{k_e Nu_a}{D_e} \]  

(A-31)

\[ Nu_a = Nu_s \left\{ 1 + \left[ 2.64 \left( \frac{\varepsilon}{D_e} \right)^{0.212} \left( \frac{p}{D_e} \right)^{-0.21} \left( \frac{\alpha}{90} \right)^{0.29} \left( \frac{\Pr}{0.024} \right)^{2} \right] \right\} \]  

(A-32)

\[ Nu_s = \frac{f \left( \frac{Re}{Pr} \right)}{2 \left[ 1 + 12.7 \sqrt{f / 2 \left( \Pr^{2/3} - 1 \right)} \right]} \]  

(A-33)

Where:

\[ f = \left( 1.58 \ln Re - 3.28 \right)^{-2} \]  

(A-34)

The index \( a \) refers to twisted tube, \( s \) index for smooth tube, and \( f \) the friction coefficient.

The inner wall

The equation of conservation of energy applied to the inner wall can be written assuming a constant thermal conductivity:

\[ \rho_p C_p \frac{\partial T_e}{\partial t} - k_p \frac{\partial^2 T_e}{\partial z^2} = -\frac{\pi D_i h_{sp}}{A_p} \left(T_p - T_e\right) + \frac{4D_p h_{sp}^2}{A_p} \left(T_e - T_p\right) \]  

(A-35)

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle4.com/review-history/61067