Amount of Information and Measurement Uncertainty

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

ABSTRACT

Aims: To acquaint specialists in the field of physics and technology, experimenters and theoreticians with the possibilities of using information theory to analyze the results of an experiment, without a statistical and subjective expert approach.

Place and Duration of Study: Mechanical & Refrigeration Consultation Expert, between December 2019 and February 2020.

Methodology: Using the information approach and calculating the amount of information contained in the model of measuring a physical constant, we formulate a quantitative indicator for analyzing the results of the experiment.

Results: The appropriateness of applying the described approach is checked when studying the database when measuring various physical constants. The approach is applicable to the analysis of results obtained both for a long and a short period of time.

Conclusion: The information-theoretical approach allows us to formulate a universal indicator of the threshold mismatch between the model and the phenomenon, applicable to all scientific and technical fields in which the International System of Units (SI) is used.

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1. INTRODUCTION

Currently, there is a constant increase in research and development costs, which leads to an increase in the number of researchers, the number of which for 2018 was from 7 to 8 million [1]. Each researcher must prepare and publish a certain number of articles to justify the place where he works and to confirm his scientific significance. As a result, around 2,300,000 scientific and technical articles were published in 2016 [2]. The situation that has arisen causes a certain concern by the scientific community in the veracity, validity and ability to reproduce the declared results [3,4,5,6,7,8]. In particular, scientists from the Czech Republic and Canada are ironic over the activities of chemists and materials scientists in attempts to alloy graphene with the help of many different elements and compounds under various conditions on a variety of reactions [9]. They estimated that about 2 million articles can be published on this topic! And all this—without considering various modification conditions. The authors decided to approach a parody of publications on doping graphene in all seriousness: they doped graphene with bird droppings (guano). With a fair amount of sarcasm, the following conclusion of the ‘guan’ studies of the authors was formulated: judging by the results obtained, any guano added to graphene will enhance its electro catalytic properties.

Apparently, such situations are present in different scientific disciplines. For example, there was found that at least 50% of life science research cannot be replicated [10]. The same holds for 51% of economics papers [11]. In addition, in [12] it was noted that out of more than 1,500 articles published in the International Journal of Refrigeration in the period 2013–2018, the authors of only 20% of the considered articles decided that it is necessary to compare the achieved experimental uncertainty (EU) and the difference between theoretical data (TD) and experimental data (ED). If \( EU > |TD − ED| \), the value of the proposed model is insignificant, and putting it into practice is very risky. Thus, the indicated correlation between experimental and calculated data does not guarantee that the choice of the structure of the model will be quite complete. A small absolute percentage difference between the experiment and the simulation, called by the authors of different publications ‘the accuracy of the model is in the range of 5–10–15%’, allows them to declare good agreement of the theory and experiment. However, the ‘statistical significance’ between theoretical and experimental data is not sufficient evidence of the correctness of the chosen model. A necessary condition for the accuracy of the model is, first of all, the smallness of the calculated general uncertainty of the objective function in comparison with the gap between theory and experiment. This fundamental truth has not occupied an important place in the engineering and physical literature but should be the subject of serious discussion. Unfortunately, the situation is such that the author’s citation index is growing, and the practical advantages of their research are doubtful.

In psychological science, far from metallography, life science, economics, and refrigeration, the situation may also not seem the best. In [13] an attempt was made to assess reproducibility in psychological science. The authors used five indicators to assess the reproducibility of 100 submitted works published in prestigious journals. The conclusion was unequivocal: an attempt to repeat the original results yielded weaker results: only 36% of the repetitions did the same, and the observed effects were half the estimates obtained in the original studies.

At the same time, a number of scientists [3] express an optimistic opinion, stating that the presence and identification of possibly erroneous conclusions is useful because it shows that the community is alert and involved in self-criticism and sustainable re-analysis of data, which is a hallmark of the scientific method. In some cases, ‘irreproducible data’ can be considered as part of scientific progress and stimulate the growth of scientific knowledge. Scientists [3] confirmed their conclusions by analysing experimental data on the measurement of the gravitational constant, the speed of light and the Planck constant.

The presented points of view certainly do not fully reflect the existing situation, although they can be considered, to some extent, as alarming calls about the current trend. At the same time, the author of this article, being a convinced practitioner, do not share the bravura optimism that ‘lack of reproducibility is not necessarily bad news; it may herald new discoveries and signal...
2. PREPOSITIONS

Any model contains dimensional and dimensionless variables taken from SI. In addition, the observer, based on his own experience, intuition and accumulated knowledge, by default selects the base quantities (L is the unit of length - meter (m), M is the unit of mass - kilogram (kg), T is the unit of time - second (s), I is the unit of electric current - ampere (A), θ is the unit of thermodynamic temperature - kelvin (K), F is the amount of substance (mol), and J is the unit of luminous intensity - candela (cd)) to describe the measurement process. Thus, each model can be assigned to a specific class of phenomena (CoP\textsubscript{SI}). For example, the measurement of the Planck constant is usually carried out using CoP\textsubscript{SI} ≡ LMTF or CoP\textsubscript{SI} ≡ LMTI. This means that the model uses derived variables with the dimensions of four base quantities: L, M, T, F or L, M, T, I. Although SI does not exist in nature, it is widely used by the scientific community in any research.

In addition, by calculating the number of dimensionless criteria contained in SI (μ\textsubscript{SI} = 38,265 [14], one can calculate the initial smallest absolute uncertainty Δ\textsubscript{pmm} inherent in the model and the comparative uncertainty [15] due to the choice of CoP\textsubscript{SI} and the number of recorded variables during a model formulation and before computer calculations or experiment

\[ \Delta_{\text{pmm}} / S = [(z' - \beta') / \mu_{\text{SI}} + (z' - \beta') / (z' - \beta')] \]

where S is the interval in which the dimensionless quantity u is located, z' and β' are the total number of dimension variables and the number of base quantities in the CoP, respectively, z" and β" are the number of all and base quantities registered in the chosen model, respectively, k is the Boltzmann constant, \( \varepsilon = \Delta_{\text{pmm}} / S \) is the comparative uncertainty [15].

This approach is called the information-theoretic approach or \( \mu \)-rule. Two features of the \( \mu \)-rule should be noted. First of all, this equation is true for all other measurement systems. Models formulated in other systems of units of measure, for example, in centimeter–gram–second (CGS), will also have to comply with Equation (6) to maintain the basic relationships between physical variables. Equivalence ensures that physical models of reality remain consistent, regardless of units.

Using Equation (1), one can find the necessary conditions for approaching the smallest relative and comparative uncertainties of each CoP, the fulfillment of which can confirm the legitimacy of the declared measured value of the physical constant. For this, it is necessary to take the derivative of \( \Delta_{\text{pmm}} / S \) with respect to \( z' - \beta' \) and equate it to zero [16]. In Table 1 there are introduced achievable comparative uncertainties \( \varepsilon_{\text{CoP}} \) and the recommended number of dimensionless criteria \( Y_{\text{CoP}} \) corresponding to different CoP (Table 1).

<table>
<thead>
<tr>
<th>CoP\textsubscript{SI}</th>
<th>Comparative uncertainty</th>
<th>Number of criteria, ( Y_{\text{CoP}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMT</td>
<td>0.0048</td>
<td>( \geq 2 )</td>
</tr>
<tr>
<td>LMTF</td>
<td>0.0146</td>
<td>( \geq 6 )</td>
</tr>
<tr>
<td>LMTI</td>
<td>0.0245</td>
<td>( \geq 19 )</td>
</tr>
<tr>
<td>LMT8</td>
<td>0.0442</td>
<td>( \geq 52 )</td>
</tr>
<tr>
<td>LMT8F</td>
<td>0.1333</td>
<td>( \geq 169 )</td>
</tr>
<tr>
<td>LMT8I</td>
<td>0.2220</td>
<td>( \geq 471 )</td>
</tr>
<tr>
<td>LMT8IFI</td>
<td>0.6665</td>
<td>( \geq 4,249 )</td>
</tr>
</tbody>
</table>

scientific progress' [3] because their number is growing at an alarming rate. That is why an attempt to propose to the scientific community a universal theoretically substantiated criterion for assessing the threshold mismatch between the model and the observed phenomenon/object seems very urgent [14]. This criterion can be used in any scientific and technical discipline where the International System of Units (SI) is used. In addition, any implementation of the reproducibility of published research results becomes objective in nature, independent of the subjective position of experts.
Two methods are used to calculate $S$. In the first case, the possible interval of placement of the physical constant $S$ is selected as the difference between its maximum and minimum values measured by various scientific groups. It is called the Information Approach - Relative Uncertainty (IARU). Its detailed step-by-step procedure is presented in [16]. According to the second method, called the Information Approach - Comparative Uncertainty (IACU) [17], $S$ is determined by the limits of the measuring instruments used [15]. Thus, an amazing opportunity arises to apply the concepts and mathematical apparatus of information theory to calculate the value of the recommended relative uncertainty, which, in turn, allows you to declare the validity of the measured or calculated studied variable, for example, the value of a physical constant.

3. RESULTS

As a clear example of the application of the information-theoretical approach, experimental and calculated data for the period 2000–2018 are presented for measuring, using various methods, the Planck constant and the Newtonian gravitational constant (similarly [3]), as well as the Boltzmann constant and the Hubble constant (Table 2), in which there are used the following acronyms:

1. KB – Kibble balance. Data include results of measurements taken in seven laboratories from 2014 to 2018. 2. XRCD – X-ray crystal density. Data include results of measurements taken in seven laboratories from 2011 to 2018. 3. AGT – acoustic gas thermometer. Data include results of measurements taken in seven laboratories from 2009 to 2017. 4. DCGT – dielectric constant gas thermometer. Data include results of measurements taken in six laboratories from 2012 to 2018. 5. JNT – Johnson noise thermometer. Data include results of measurements taken in six laboratories from 2011 to 2017. 6. DBT – Doppler broadening thermometer. Data include results of measurements taken in six laboratories from 2007 to 2015. 7. BDL – the brightness of the distance ladder. Data include results of measurements taken in seven laboratories from 2011 to 2019. 8. CMB – cosmic microwave background. Data include results of measurements taken in six laboratories from 2009 to 2018. 9. BAO – baryonic acoustic oscillations. Data include results of measurements taken in four laboratories from 2000 to 2014. 10. Data include results of measurements taken in seven laboratories from 2000 to 2014. 11. Data include results of measurements taken in five laboratories from 2001 to 2018.

The reasoned conclusions based on the analysis of the data of Table 2 and which, at times, elude the attention of the scientific community, can be summarized as follows:

3.1 Planck Constant, $h$

Test benches and the measurement procedure for $h$ are implemented using the following classes: $LMT_1$ and $LMT_2$. The ratio $r_{exp/SI} = 2.9$ when using the Kibble balance method ($CoP_{SI} \equiv LMT_1$) is much smaller than when implementing the X-ray crystal density (XRCD) method ($r_{exp/SI} = 9.1$). That is why, even though the value of comparative uncertainty in XRD is less (0.0245 > 0.0145), it is urgent to reduce the influence of sources of uncertainty for XRCD. Because the experimental relative uncertainties are almost equal (1.3 $\cdot$ 10$^{-6}$ = 1.2 $\cdot$ 10$^{-6}$), it can be argued that the Kibble balance method is preferable when measuring $h$ and takes into account a larger number of potential effects of interaction between variables.

3.2 Boltzmann Constant, $k_b$

Measuring of $k_b$ is organized by the use of two $CoP_{SI}$: $LMT_9F$ and $LMT_1$. From the data in Table 1, it can be seen that the ratios $r_{exp/SI}$ (1.9) and (1.1) achieved using JNT and DBT suggest that these methods achieved high accuracy in measuring $k_b$ in 2009–2018. In addition, following the $\mu$-rule, it should be noted that the achieved experimental least relative uncertainty of 3.7 $\cdot$ 10$^{-7}$ during DCGT is doubtful. This is explained by the requirement of the $\mu$-rule, according to which the theoretically calculated relative uncertainty (4.3 $\cdot$ 10$^{-7}$) is always less than the experimental uncertainty (3.7 $\cdot$ 10$^{-7}$). Therefore, researchers using DCGT should reanalyse all possible sources of uncertainty.

3.3 Hubble Constant, $H_0$

To calculate $H_0$, researchers use two $CoP_{SI}$: $LMT$ and $LMT_0$. The experimental relative uncertainties of 0.01 when using BDL and BAO ($CoP_{SI} \equiv LMT$), calculated in accordance with IARU, are many times higher than the recommended 0.00023 and 0.00018,
respectively. This means that many hidden variables were not considered in the calculations, and $\text{CoP}_{\text{SI}} \equiv \text{LMT}$ cannot be recommended in the future. Therefore, the conviction of scientists in accounting for all possible sources of uncertainties is far from guaranteeing the achievement of the true value of $H_0$ by these two methods.

For BDL and BAO, the $r_{\text{exp}}/r_{\text{SI}}$ ratio is 43 and 56, respectively, although when measuring $H_0$ with CMB, $r_{\text{exp}}/r_{\text{SI}} = 2.4$. This indicates a situation where, according to most astronomers using various methods of calculating $H_0$, an increase in the number of observed space objects creates the illusion of ideal statistical stability. However, starting from a certain critical amount of data, the decrease in the level of fluctuations stops. A further increase in the amount of data either practically does not affect the level of fluctuations in the estimates, or even leads to their growth [20]. That is why, following the logic of the information approach, the method of measuring $H_0$ using the cosmic microwave background is the most promising, theoretically substantiated and implementing the most reliable experimental data.

### 3.4 Newtonian Gravitational Constant, $G$

$G$ measurements are followed by $\text{CoP}_{\text{SI}} \equiv \text{LMT}$ (mechanical methods) and $\text{CoP}_{\text{SI}} \equiv \text{LMTI}$ (electro-mechanical methods). The huge difference in $r_{\text{exp}}/r_{\text{SI}}$ ratios (12.7 and 1.9, respectively) confirms the thesis of the information approach about the inappropriateness of using mechanical methods to measure the true value of the gravitational constant.

In the framework of the information-theoretical approach, further refinement of the true value of the gravitational constant and a decrease in the experimental relative uncertainty is possible using models and measurement methods with a large number of base quantities, for example, $\text{CoP}_{\text{SI}} \equiv \text{LMTI}$.

In general, the data presented (Table 2) allow us to formulate specific conclusions and comments on the assessment of the achieved measurement accuracy at the present stage of development of science and technology. Obviously, some of them do not coincide with generally accepted provisions of the scientific community, in particular, in experimental physics and cosmology.

1. The ratio $r_{\text{exp}}/r_{\text{SI}}$ is calculated in accordance with the IACU and is called the ‘coefficient of consistency’ for the physical constant, as measured by various methods. This coefficient is an objective criterion for establishing the acceptability of a particular measurement method and assessing the achieved accuracy when comparing different measurement methods for one specific physical constant.

This is confirmed by the following considerations. When measuring $H_0$, the ratio $r_{\text{exp}}/r_{\text{SI}}$ is 710 (BDL) and 104 (BAO), and for $G$ measured using mechanical methods, $r_{\text{exp}}/r_{\text{SI}} = 100$. At the same time, when measuring $H_0$ by electro-mechanical methods, $r_{\text{exp}}/r_{\text{SI}} = 7.9$, and when measuring the Planck constant with KB and XRCD and using AGT and DCGT to calculate the Boltzmann constant, the values of the $r_{\text{exp}}/r_{\text{SI}}$ ratios are very close to each other. This situation indicates that BDL, BAO for $H_0$, and mechanical methods for $G$ have limited use. It is important to emphasize that, using the IACU, researchers can find out for which method of measuring the physical constant it is necessary to continue the search for all possible sources of uncertainties.

2. The $r_{\text{exp}}/r_{\text{SI}}$ ratio is calculated taking into account the IARU methodology and allows us to identify a clearly manifested trend (Table 1): models of measurements of physical constants with a small number of base quantities ($\text{LMT}$ and $\text{LMTI}$) have clearly overestimated values of this ratio: 9.1, 12.7, 44 and 56. This is due to ignoring the influence of unaccounted basic quantities and possible relationships between variables in calculating the value of a physical constant. Moreover, for models with a large number of base quantities, for example, $\text{LMTI}$, $\text{LMTI}_F$, $\text{LMTB}$ or $\text{LMTB}_F$, the ratio $r_{\text{exp}}/r_{\text{SI}}$ varies from 0.9 to 2.9. Thus, as part of the information approach, we can consider the $r_{\text{exp}}/r_{\text{SI}}$ ratio as a universal indicator for establishing the reliability of measurement methods and the reliability of the achievements of scientists working in various research centres in measuring any physical constant using various methods. Moreover, the assessment of the value (credibility) of the obtained experimental results does not require any statistical combination of the results of all participating laboratories.

3. The minimum achievable relative uncertainties, $r_{\text{SI}}$, calculated in accordance with the information approach, for different methods of measuring even the same physical constant are very different from each other. So, for the
Table 2. Generalized data on the measurement of physical constants by various methods*

<table>
<thead>
<tr>
<th>Physical constant/ Publications interval</th>
<th>Planck constant, ( h ) 2009–2017</th>
<th>Boltzmann constant, ( k_b ) 2009–2018</th>
<th>Hubble constant, ( H_0 ) 2009–2019</th>
<th>Gravitational constant, ( G ) 2000–2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement method</td>
<td>KB(^1)</td>
<td>XRC(^2)</td>
<td>AGT(^3)</td>
<td>DCGT(^4)</td>
</tr>
<tr>
<td></td>
<td>XRCD(^5)</td>
<td>AGT(^6)</td>
<td>DCGT(^7)</td>
<td>JNT(^8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>DBT(^9)</td>
<td>BDL(^{10})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>CMB(^{11})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>BAO(^{12})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mechanical methods(^{10})</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Electro-mechanical methods(^{11})</td>
</tr>
<tr>
<td>CoP</td>
<td>LMT(^{13})</td>
<td>LMT(^{14})</td>
<td>LMT(^{15})</td>
<td>LMT(^{16})</td>
</tr>
<tr>
<td>Comparative uncertainty according to CoP(<em>{Si}), ( \varepsilon</em>{Si}^{**} )</td>
<td>0.0245</td>
<td>0.0145</td>
<td>0.1331</td>
<td>0.2220</td>
</tr>
<tr>
<td>Achieved experimental lowest comparative uncertainty ( (IACU)), ( \varepsilon_{exp}^{**} )</td>
<td>0.3976</td>
<td>0.4733</td>
<td>0.4832</td>
<td>0.5044</td>
</tr>
<tr>
<td>Ratio ( \varepsilon_{exp}/\varepsilon_{Si}^{**} )</td>
<td>15.9</td>
<td>32.6</td>
<td>3.6</td>
<td>2.3</td>
</tr>
<tr>
<td>Relative uncertainty according to CoP(<em>{Si}) ( (IARU)), ( r</em>{Si}^{**} )</td>
<td>4.5\cdot10^{-9}</td>
<td>1.0\cdot10^{-8}</td>
<td>2.3\cdot10^{-7}</td>
<td>4.3\cdot10^{-7}</td>
</tr>
<tr>
<td>Achieved experimental lowest relative uncertainty, ( r_{exp}^{**} )</td>
<td>1.3\cdot10^{-8}</td>
<td>9.1\cdot10^{-8}</td>
<td>6.0\cdot10^{-7}</td>
<td>3.7\cdot10^{-7}</td>
</tr>
<tr>
<td>Ratio ( r_{exp}/r_{Si}^{**} )</td>
<td>2.9</td>
<td>9.1</td>
<td>2.6</td>
<td>0.9</td>
</tr>
</tbody>
</table>

* Data are introduced in [16,17,18,19]
Boltzmann constant, the calculated relative uncertainties, \( r_{si} \), differ by two orders of magnitude, and for the Hubble constant, by an order of magnitude. Therefore, in the framework of the information approach, in contrast to the concept approved by the Committee on Data for Science and Technology, it is not recommended to declare only one value of relative uncertainty when measuring the physical constant by various methods.

Moreover, the assessment of the value (reliability) of the obtained experimental results does not require any statistical combination of the results of all participating laboratories. Thus, in the framework of the information approach, bright hopes that the mutual agreement of the results of different research laboratories using fundamentally different measuring methods justifies the assumption of the reliability of the data [3] are unlikely to materialize and can hardly be justified.

4. CONCLUSIONS

It is important to pay attention on the topic stated in the title of this article. It is clear that it is difficult to come to conclusions that are so radically different from the point of view of many scientists if one uses the same methodology as they do. The difference lies primarily in the fact that the researchers who formed the traditional statistical and expert approach to the analysis of experimental data, in particular, the results of measurements of physical constants, completely relied on statistics obtained after formulation of a model, which are subject to the threatening influence of various sources of uncertainties. At the same time, three rather powerful corpora of evidence preceding any experiments were not taken into account at once, namely: A qualitative set of variables in the model—the class of the phenomenon, the number of variables taken into account and the uncertainty that already exists in formulating the model. Thus, considering these three reasons, it seems that the information-theoretical approach is alone a universal tool for establishing the soundness of measurement methods and increasing the trustworthiness of scientific results.

The conscious choice of one or another class of phenomena plays a role that can no longer be ignored. This is simply an inevitable effect that you have to put up with when modeling the observed process. All researchers modeling natural and technological processes are limited in predicting the measurement results with high accuracy, which is declared in the \( \mu \)-rule.

ETHICAL APPROVAL

The author confirms that this study is not against the public interest, or that the release of information is allowed by legislation.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES


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