Kink Type Traveling Wave Solutions of Right-handed Non-commutative Burgers Equations via Extended \((G'/G)\)-Expansion Method

Harun-Or-Roshid

1Department of Mathematics, Pabna University of Science and Technology, Bangladesh.

Author's contribution

This whole work was carried out by the author HOR.

Article Information

DOI: 10.9734/PSIJ/2019/v21i43017

(1) Dr. Christian Brosseau, Distinguished Professor, Department of Physics, Université de Bretagne Occidentale, France.

(2) Ahmad Neirameh, Gonbad University, Iran.

Complete Peer review History: http://www.sdiarticle3.com/review-history/12512

Original Research Article

ABSTRACT

The extended \((G'/G)\)-expansion method is significant for finding the exact traveling wave solutions of nonlinear evolution equations (NLEEs) in mathematical physics. In this paper, we enhanced new traveling wave solutions of right-handed non-commutative burgers equations via extended \((G'/G)\)-expansion. Implementation of the method for searching exact solutions of the equation provided many new solutions which can be used to employ some practically physical and mechanical phenomena. Moreover, when the parameters are replaced by special values, the well-known solitary wave solutions of the equation rediscovered from the traveling wave solutions and included free parameters may imply some physical meaningful results in fluid mechanics, gas dynamics, and traffic flow.

Keywords: The extended \((G'/G)\)-expansion method; the right-handed non-commutative burgers equation; solitons wave solutions; traveling wave solutions.


*Corresponding author: E-mail: harunoroshidmd@yahoo.com;
1. INTRODUCTION

Most of the complex nonlinear phenomena in plasma physics, fluid dynamics, chemistry, biology, mechanics, elastic media and optical fibers etc. can be explained by nonlinear evolution equations (NLEEs) [1]. When we want to understand the physical mechanism of the phenomena, exact solutions have to be explored. So, investigations of exact traveling wave solutions play a vital role in applied mathematics, physics and engineering branches in the study of complex physical and mechanical phenomena. Recently, a number of prominent mathematicians and physicists have worked out on this interesting area of research to obtain exact solutions of NLEEs using symbolic computer programs such as Maple, Matlab, Mathematica that facilitate complex and tedious algebraical computations [2,3].

A lot of physical models have supported a wide variety of solitary wave solutions to realize the internal physical mechanism for example, the wave phenomena observed in fluid dynamics [4,5], plasma and elastic media [6,7] and optical fibers [8,9] etc. Some of the existing powerful methods for deriving exact solutions of NLEEs are Backlund transformation method [10], Darboux Transformations [11], tanh-function method [12], Exp-function method [13], the first integral method [14] and so on. Wang et al. [15] firstly proposed the \((G'/G)\)-expansion method, then many diverse group of researchers extended this method by different names like extended \((G'/G)\)-expansion method [2,3,16,17], generalized \((G'/G)\)-expansion method [18], modified simple equation method [19-21] with different auxiliary equations. Zayed [22] established extended \((G'/G)\)-expansion method for solving the (3+1)-dimensional NLEEs in mathematical physics [23,24]. Recently, Khan et al. [25] found traveling and soliton wave solutions of GZK-BBM and right-handed non-commutative burgers equations by Modified Simple Equations method.

In this article, our motivation is to add new more general traveling wave solutions of right-handed non-commutative burgers equations via extended \((G'/G)\)-expansion. The performances of the method will encourage other researchers to apply it in other nonlinear evolution equations [26,27].

2. MATERIALS AND METHODS

For given nonlinear evolution equations with independent variables \(x\) and \(t\), we consider the following form

\[
F(u, u_x, u_{xx}, u_{xxx}, \ldots) = 0
\]  

(1)

By using traveling wave transformation

\[
u(x, t) = u(\xi), \quad \xi = x - V t
\]  

(2)

where \(u\) is an unknown function depending on \(x\) and \(t\), and is a polynomial \(F\) in \(u(x, t) = u(\xi)\) and its partial derivatives and \(v\) is a constant to be determined later. The existing steps of method are as follows:

Step 1: Using the Eq.(2) in Eq.(1), we can convert Eq. (1) to an ordinary differential equation

\[
Q(u, -vu', u', -vu', \ldots) = 0
\]  

(3)

Step 2: Assume the solutions of Eq.(3) can be expressed in the form

\[
u(\xi) = \sum_{i=-n}^{n} [a_i (G'/G)^i + b_i (G'/G)^{i+1}] \sqrt{\sigma[1+(G'/G)^i]} \]  

(4)

with \(G = G(\xi)\) satisfying the differential equation

\[
G' + \mu G = 0
\]  

(5)

in which the value of \(\sigma\) must be \(\pm 1, \mu \neq 0\), \(a_i, b_i\), \((i = -n, \ldots, n)\) and \(\lambda\) are constants to be determined later. We can evaluate \(n\) by balancing the highest- order derivative term with the nonlinear term in the reduced equation (3).

Step 3: Inserting Eq.(4) into Eq.(3) and making use of Eq.(5) and then extracting all terms of like powers of \((G'/G)^i\) and \((G'/G)^{i+1} \sqrt{\sigma[1+(G'/G)^i]}\)

\[
\text{together set each coefficient of them to zero yield a over-determined system of algebraic equations and then solving this system of algebraic equations for } a_i, b_i, \ (i = -n, \ldots, n) \text{ and } v, \text{ we obtain several sets of solutions.}
\]

Step 4: For the general solutions of Eq.(5), we have

\[
\mu < 0, \quad \frac{G'}{G} = \sqrt{-\mu} \left[ \frac{A \sinh(\sqrt{-\mu} \xi) + B \cosh(\sqrt{-\mu} \xi)}{A \cosh(\sqrt{-\mu} \xi) + B \sinh(\sqrt{-\mu} \xi)} \right] = f(\xi)
\]  

(6)
where $A$, $B$ are arbitrary constants. At last, inserting the values of $a_i, b_i, (i = -n, ..., n)$, $v$ and (6,7) into Eq. (4) and obtain required traveling wave solutions of Eq. (1).

3. APPLICATION OF OUR METHODS

In this section, we will bring to bear the extended \((G'/G)^N\) -expansion method to find the traveling wave solutions to the right-handed nc-Burgers equation:

\[
u_{xx} + 2uu' + vu' = 0 \quad (9)
\]

Integrating (9) with respect to $\xi$ and setting the constant of integration to zero, we obtain

\[
u' + u^2 + vu = 0 \quad (10)
\]

Balancing the highest order derivative and nonlinear term, we obtain $N = 1$. Thus, the trial solutions of the Eq. (10) takes the form

\[
u(\xi) = a_0 + a_1(G'/G) + a_1(G'/G)^{-1} + b_0(G'/G)^2[1 + (G'/G)'^2 / \mu] \quad (11)
\]

where $G = G(\xi)$ satisfies Eq.(5). Substituting Eq. (11) and Eq. (5) into Eq. (10), collecting all terms with the like powers of $(G'/G)'$ and $(G'/G)^2[1 + (G'/G)'^2 / \mu]$, and setting them to zero, we obtain a over-determined system that consists of fourteen algebraic equations.

Solving this over-determined system with the assist of Maple and inserting in eq (11), we have the following results.

Case-1: $v = \pm 2\sqrt{\mu}, a_0 = -\mu, a_1 = \sqrt{-\mu}, a_i = b_i = b_0 = b_1 = 0$. Now when $\mu > 0$, then using (7) and (11), we have

\[
u_1(\xi) = \sqrt{-\mu - \mu(f_1(\xi))^{-1}}, \text{ where } \xi = x + 2\sqrt{-\mu} \quad (12)
\]

\[
u_1(\xi) = -\sqrt{-\mu - \mu(f_1(\xi))^{-1}}, \text{ where } \xi = x - 2\sqrt{-\mu} \quad (13)
\]

and when $\mu < 0$, then using (6) and (11), we have

\[
u_1(\xi) = \sqrt{-\mu - \mu(f_1(\xi))^{-1}}, \text{ where } \xi = x + 2\sqrt{-\mu} \quad (14)
\]

\[
u_1(\xi) = -\sqrt{-\mu - \mu(f_1(\xi))^{-1}}, \text{ where } \xi = x - 2\sqrt{-\mu} \quad (15)
\]
Case-2: \( v=\sqrt{4\mu} a_{1} = -\mu a_{1} = 2\sqrt{\mu} a_{1} = 1, b_{1} = b_{0} = 0 \).

Now when \( \mu > 0 \), then using (7) and (11), we have
\[
u_{1}(\xi) = 2\sqrt{\mu} + f_{1}(\xi) \sqrt{\mu} f_{1}(\xi), \quad \text{where} \quad \xi = x + 4\sqrt{\mu} \mu
\]  \hspace{1cm} (16)
and when \( \mu < 0 \), then using (6) and (11), we have
\[
u_{1}(\xi) = -2\sqrt{\mu} + f_{1}(\xi) \sqrt{-\mu} f_{1}(\xi), \quad \text{where} \quad \xi = x - 4\sqrt{\mu} \mu
\]  \hspace{1cm} (17)

Case-3: \( v=\pm 2\mu a_{1} = -\mu a_{1} = \mp 1, b_{1} = b_{0} = 0 \).

Now when \( \mu > 0 \), then using (7) and (11), we have
\[
u_{1}(\xi) = \sqrt{\mu} + f_{1}(\xi), \quad \text{where} \quad \xi = x + 2\sqrt{\mu} \mu
\]  \hspace{1cm} (20)
and when \( \mu < 0 \), then using (6) and (11), we have
\[
u_{1}(\xi) = -\sqrt{\mu} + f_{1}(\xi), \quad \text{where} \quad \xi = x - 2\sqrt{-\mu} \mu
\]  \hspace{1cm} (21)

Case-4: \( v=\pm 2\mu a_{1} = -\mu a_{1} = \mp 1/2, b_{1} = b_{0} = 0 \).

Now when \( \mu > 0 \), then using (7) and (11), we have
\[
u_{1}(\xi) = \sqrt{\mu} / 2 \mp f_{1}(\xi) / 2 \pm \sqrt{\mu} / 4\sigma f_{1}(\xi) / \mu, \quad \text{where} \quad \xi = x \pm 2\sqrt{\mu} \mu
\]  \hspace{1cm} (24)
and when \( \mu < 0 \), then using (6) and (11), we have
\[
u_{1}(\xi) = -\sqrt{\mu} / 2 \mp f_{1}(\xi) / 2 \pm \sqrt{\mu} / 4\sigma f_{1}(\xi) / \mu, \quad \text{where} \quad \xi = x \mp 2\sqrt{-\mu} \mu
\]  \hspace{1cm} (25)

Case-5: \( v=\pm 2\mu a_{1} = 1, b_{1} = b_{0} = 0 \).

Now when \( \mu > 0 \), then using (7) and (11), we have
\[
u_{1}(\xi) = \mu_{1}(\xi) = -\sqrt{\mu} / 2 - \mu f_{1}(\xi) / 2 \pm \mu \sqrt{4\sigma f_{1}(\xi) / \mu}, \quad \text{where} \quad \xi = x + \sqrt{-\mu} \mu
\]  \hspace{1cm} (28)

Remark 1: It is shown that if we put \( \frac{d}{\mu} = 0 \) and \( \sqrt{-\mu} = -\sqrt{\mu} \), we obtain the soliton solution (36) in Ref.[25] and if we put \( \mu = 0 \) and \( \sqrt{-\mu} = -\sqrt{-\mu} \), we obtain the soliton solution (37) in Ref.[25].

Remark 2: We have verified all the achieved solutions by putting them back into the original equation (8) with the aid of Maple 13.

4. GRAPHICAL REPRESENTATIONS OF OUR SOLUTIONS

The graphical illustrations of the solutions are depicted in the Figs. 1 to 4 with the aid of commercial software Maple 13. Solutions \( u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}, u_{7}, u_{8}, u_{9}, u_{10}, u_{11}, u_{12}, u_{13} \), and \( u_{14} \) are similar to the figure of solution \( u_{1} \), the shape of the exact singular Kink solution of \( u_{1} \), and \( u_{1} \), respectively. The shape of figures of solutions \( u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{6}, u_{7}, u_{8}, u_{9}, u_{10}, u_{11}, u_{12}, u_{13}, u_{14} \), and \( u_{14} \) are similar to the figure of solution \( u_{14} \), and the shape of figure of solution \( u_{14} \) is similar to the figure of solution \( u_{14} \) and (18) and so the figures of these solutions are omitted for convenience. Solution \( u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{20} \) describes the kink wave. Kink waves are traveling waves which arise from one asymptotic state to another. The kink solutions are approach to a constant at infinity. Fig. 4 shows the shape of the exact Kink-type solution of \( u_{16} \) the right-handed noncommutative burgers equation (8).

The shape of figures of solutions \( u_{11}, u_{12}, u_{13}, u_{14} \), and \( u_{15} \) are similar to the figure of solution of \( u_{16} \) and so the figures of these solutions are omitted for convenience.
5. CONCLUSION

The extended \((G'/G)\)-expansion method has been successfully implemented to search exact traveling wave solutions for the right-handed non-commutative burgers equation. As a result, we obtained plentiful new and fresh exact solutions which might have significant impact on future researches. The method offers solutions with free parameters that might be important to explain some intricate physical phenomena. Some special solutions including the known solitary wave solution are originated by setting appropriate values for the parameters. Overall, the results reveal that this method is productive, effective and well-built mathematical tool for solving nonlinear evolution equations and can be used in many other nonlinear evolution equations.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES


© 2019 Roshid; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sdiarticle3.com/review-history/12512